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Behaviour of Multi-Degree-of-Freedom Shear Structure Isolated Using VFPI

Sinha, Ravi[1] and Pranesh, M.[2]

ABSTRACT

The friction pendulum system (FPS) is an effective vibration control system incorporating isolation, energy dissipation and restoring mechanism in one unit. In this paper, new isolation system called as Variable Frequency Pendulum Isolator (VFPI) is examined. The VFPI exhibits progressive period shift with sliding displacement, leading to more effective isolation for large base excitations. A five-storey shear structure subjected to El Centro ground motion is analysed to illustrate the effectiveness of VFPI. The results are compared with those obtained using FPS. It is found that VFPI has superior performance when compared to the FPS isolator.

1 INTRODUCTION

Aseismic design of a structure using base isolation systems has been an area of active research [Kelly 1986]. Among the different isolation systems, the use of friction-type base isolators have been found to be very effective in reducing the acceleration response of a structure, and incorporate isolation and energy dissipation in one unit [Mostaghel et al. 1983]. The performance of these isolators is also relatively insensitive to severe variations in the frequency content and amplitude of input excitation [Mostaghel and Tanbakuchi 1983]. However, the use of simple sliding isolator (pure-friction system) may result in large sliding and residual displacements even for moderate ground motions, which are difficult to incorporate in design. This difficulty can be eliminated through the introduction of a proper restoring mechanism that tends to bring the structure back to its original position at the end of the ground motion.

A modified isolation system called the friction-pendulum system (FPS) has been recently proposed in which restoring mechanism is provided by gravity [Zayas et al. 1990; Mokha et al. 1991]. The schematic diagram of this system is shown in fig. 1. In this system, the sliding and re-centring mechanisms are integrated in one unit wherein the sliding surface takes a concave spherical shape. The FPS isolators have many advantages over the traditional friction isolator; however, severe practical difficulties of aseimsic design using FPS isolator have been observed due to its spherical shape and corresponding fixed time-period [Pranesh 1998].

The authors have recently proposed a new isolator called the Variable Frequency Pendulum Isolator (VFPI), that also uses gravity for restoring mechanism and is intended to achieve (1) progressive period lengthening, and (2) restoring force softening with increase in sliding displacement [Sinha and Pranesh 1998]. VFPI has been found to be effective in protecting structures and equipment under a variety of earthquake excitations, and it overcomes most limitations of FPS Isolators [Pranesh and Sinha 1998]. In the present paper a complex modal analysis method is proposed for analysing a structure isolated by VFPI. An example structure is isolated using VFPI with different isolator properties to show the effectiveness of VFPI. The results have also been compared with that of a structure isolated by conventional FPS. It is found that the use of VFPI retains the advantages of FPS. In addition, it is also found that substantial reduction in the response can be achieved by isolating the structure with VFPI and suitable parameters of VFPI can be chosen as per the design requirements.

2 ISOLATOR GEOMETRY

Consider a rigid block of mass m sliding on a curved surface of any geometry, $y = f(x)$ representing the sliding surface of the isolator. The origin of the surface is at the centre where the sliding displacement is zero. At any instant the restoring force is given by

$$f_R = mg\frac{dy}{dx} \tag{1}$$

[1] Associate Professor, Dept. of Civil Engineering, IIT Powai, Mumbai - 400 076, India.
[2] Graduate Student, Dept. of Civil Engineering, IIT Powai, Mumbai - 400 076, India

Assuming that this restoring force is provided by an equivalent spring, the spring force can be expressed as the product of the spring stiffness and its deformation. The spring stiffness in turn may be expressed as product of mass and square of frequency. So,

$$f_R = m\omega_b^2(x)x \tag{2}$$

where $\omega_b(x)$ can be called instantaneous isolator frequency, which depends only on the geometry of sliding surface. In case of FPS isolator, this frequency is approximately a constant and the restoring force is almost linear. VFPI uses a different sliding surface, which is derived from an elliptical curve with its major axis being a linear function of sliding displacement [Sinha and Pranesh 1998]. The geometry of sliding surface is defined by

$$y = b\left[1 - \frac{\sqrt{d^2 + 2dx\,\mathrm{sgn}(x)}}{d + x\,\mathrm{sgn}(x)}\right] \tag{3}$$

Now, accordingly, the slope at any displacement can be determined from

$$\frac{dy}{dx} = \frac{bd}{(d + x\,\mathrm{sgn}(x))^2\sqrt{d^2 + 2dx\,\mathrm{sgn}(x)}}x \tag{4}$$

Defining $r = x\,\mathrm{sgn}(x)/d$ and the initial frequency when x=0 as $\omega_I^2 = gb/d^2$, we have,

$$\omega_b^2(x) = \frac{\omega_I^2}{(1+r)^2\sqrt{1+2r}} \tag{5}$$

In the above equations, b and d are the parameters that completely define the isolator properties. The ratio b/d^2 controls the initial frequency of the isolator and the value of d decides the rate of variation of the isolator frequency. The variation of the isolator frequency of VFPI and FPS with respect to the isolator sliding displacement is shown in fig. 3. It is seen that the frequency of VFPI sharply decreases with sliding displacement and achieves large period shifts for large displacements. In contrast to this the frequency of FPS isolator is almost constant. The force-deformation hesteresis loops for VFPI and FPS are shown in fig. 3. The hesteresis loop for VFPI clearly shows smooth reduction of restoring force for high isolator displacements. This property is extremely beneficial during higher intensity earthquakes due to reduction in the isolator force at large sliding displacements.

3. RESPONSE OF STRUCTURE

Consider an N-storey shear building isolated by VFPI (figure 2). Due to the action of frictional forces, the motion consists of two phases namely, non-sliding phase and sliding phase. The equations of motion are different in the two phases and the overall behaviour is highly non-linear.

3.1 Non-sliding Phase

In this phase the structural behaviour is identical to a fixed-base structure and hence the usual modal analysis can be carried out. The equations of motion in this phase are as below.

$$\mathbf{M}_0\ddot{\mathbf{x}}_0 + \mathbf{C}_0\dot{\mathbf{x}}_0 + \mathbf{K}_0\mathbf{x}_0 = -\mathbf{M}_0\mathbf{r}_0\ddot{x}_g \tag{6}$$

$$x_b = \dot{x}_b = \ddot{x}_b = 0 \tag{7}$$

with,

$$\left|\left\{\sum_{i=1}^{N} m_i(\ddot{x}_i + \ddot{x}_g) + m_b\ddot{x}_g\right\} + m_t\omega_b^2(x_b)x_b\right| < m_t\mu g \tag{8}$$

where, $\mathbf{M}_0$, $\mathbf{C}_0$ and $\mathbf{K}_0$ are the mass, damping and stiffness matrices of the fixed-base structure, respectively, $\mathbf{x}_0$ is the vector of displacements of the DOFs of the fixed-base structure relative to the base, x_b is the displacement of the base mass relative to the ground, $\mathbf{r}_0$ is the influence coefficient vector, and dot indicates derivative with respect to time. The suffix i indicates the ith degree-of-freedom of the superstructure, m_t is the total mass of the structure, μ is the coefficient of friction and g is the acceleration due to gravity. Equations 6 and 7 can be solved using standard modal analysis procedures.

3.2 Sliding Phase

Once the LHS of inequality in equation 8 exceeds the frictional force, the structure enters sliding phase and the DOF corresponding to the base is also activated. The structure starts sliding in a direction opposite to the direction of the sum of total inertia force and restoring force. The equations of motion now are given by,

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = -\mathbf{M}\mathbf{r}\ddot{x}_g - \mathbf{r}\mu_f \tag{9}$$

In the above equation **M**, **C**, **K** are the modified mass, damping and stiffness matrices including the base DOF, **x** includes the sliding displacement of the isolator, **r** is the influence coefficient vector, μ_f is the total frictional force given by,

$$\mu_f = m_t \mu g \,\mathrm{sgn}(\dot{x}_b) \tag{10}$$

All the matrices in equation 9 are of order N+1. The signum function assumes a value of +1 for positive sliding velocity and −1 for negative velocity. As the displacements at the isolator level are predominantly rigid-body motions, the normal modes of the fixed-base superstructure can be assumed to be unmodified due to sliding. However, since the modal damping in the superstructure is typically different from the damping in the isolator, the structure is non-classically damped. As a result, modal analysis of equation 9 require the solution of complex eigen-value problem [Pranesh 1998]. The isolated modes of the structure can be expressed as a linear combination of normal modes of the superstructure and the corresponding modal coordinates. If n superstructure modes are considered for analysis, then the complex modal analysis need to be carried out for n+1 DOFs only. The analysis procedure thus consists of alternately solving usual modal analysis of the structure in the non-sliding phase, and complex modal analysis of the entire structure in the sliding phase, always considering n normal modes of the superstructure. Due to the non-linear restoring force of the isolator the complex modal properties are required to be determined at each time step in the sliding phase.

3.3 Energy Balance Equation

When energy is introduced in the isolated structure due to base excitation, the structure may or may not slide depending on the characteristics of ground motion. In case the structure is in a non-sliding phase, all the energy is fed into the structure and its behaviour is identical to a fixed-base structure. When the structure is in a sliding state, the absolute input energy can be defined similar to that in a fixed-base structure as the work done by the total inertia force (which is also equal to the total isolator force) due to ground displacement [Uang and Berterro 1990]. A part of this energy is dissipated due to sliding friction in the isolator, and the remaining energy is transmitted to the structure. Of this energy, a portion is dissipated through structural damping and the remaining is the recoverable potential and kinetic energy. So, the energy balance equation can be written as given below [Pranesh 1998].

$$E_k + E_s + E_r + E_\xi + E_\mu = E_i \tag{11}$$

where E_k is the total of absolute kinetic energies of all the masses, E_s and E_r are the restorable elastic energy due to structural deformations and potential energy due to rising of the structure along the curved sliding surface of the isolator respectively, E_ξ and E_μ are the energy dissipated due to viscous damping and sliding friction respectively and E_i is the absolute input energy. These quantities can be estimated by solving the equations of motion given above.

4. EXAMPLE PROBLEM

Results from time-history analysis for a five-storey shear structure supported on VFPI subjected to NS component of El Centro 1940 ground motion are presented in this section. The earthquake ground motion is scaled to 0.5, 1.0 and 2.0 times the recorded intensity (of peak ground acceleration of 0.33g). The scale factor of 0.5 is taken to represent the ground motions due to a small earthquake, while the scale factor of 2.0 represents a severe earthquake. The response results are compared with that of the same structure isolated using FPS whose time period equal to the initial time period of the VFPI. The FPS used for the analyses has radius 1.0 metre so that its period of 2.0 seconds. The superstructure is a five-storey uniform shear building, 5-m square in plan as shown in fig.2. The masses and storey stiffness are chosen such that the fundamental frequency of the fixed-base structure is close to the predominant frequency of El Centro ground motion (approximately 2Hz). The structure is represented as a lumped mass model with equal lumped masses of 60080 kg and equal storey stiffness of 112600 kN/m. The natural frequencies and modal properties for the fixed-base and isolated structure are given in Table 1. It should, however be kept in mind that the natural frequencies of a structure isolated by VFPI change continuously with the isolator displacement and the frequencies shown in Table 1 indicate only an upper bound on the frequencies when the isolator displacement is zero. The VFPI chosen is such that it has an initial time period of 2.0 sec and the time period sharply increases to 5.0 sec in a sliding displacement of around 100 mm. The corresponding values of isolator parameters b and d are 0.01-m and 0.10-m, respectively. The analysis is carried out for three coefficients of friction, μ=0.02, μ=0.05 and μ=0.10. The structural damping is assumed as 5% of critical damping in all modes.

TABLE 1 Modal properties of fixed-base and isolated structure

Mode	Isolator	1	2	3	4	5
Fixed-base – Frequency (Hz)	-	1.96	5.72	9.02	11.59	13.22
Effective Mass Participation (%)	-	87.95	8.72	2.42	0.75	0.16
Isolated – Frequency (Hz)	0.49	3.64	6.92	9.76	11.93	13.31
Effective Mass Participation (%)	99.93	0.07	0.00	0.00	0.00	0.00

4.1 Modal contributions

The contributions of superstructure modes to the peak response and energy quantities are shown in Table 2 and 3 respectively. It is observed that only one mode of the superstructure may be adequate to calculate the response quantities at the base. However, the response calculations of the superstructure are sensitive to the contribution of more number of modes. This is because the entire isolator displacement depends on the first mode and hence the response quantities of the base are hardly affected by the higher modes. It is seen that the higher mode contributions are especially significant for the accelerations and the shears. This is due to the highly non-linear behaviour of the isolators accompanied by stick-slip motions. For a reasonable estimate of the superstructure quantities use of 2-modes of the superstructure may be enough. However, the analysis with one mode still gives good approximation of the response quantities and may be used for preliminary design as the shears are overestimated. As the accelerations are significantly underestimated by the first mode, larger number of modes are required to be considered.

TABLE 2 Modal contributions of response quantities

No. of Superstructure Modes	**1**	**2**	**3**	**4**	**5**
Absolute accl. of top (g)	0.1388	0.1381	0.1779	0.1715	0.1698
Isolator displacement (m)	0.3861	0.3848	0.3813	0.3817	0.3817
Base shear (kN)	189.70	153.69	156.94	153.08	152.77

TABLE 3 Modal contributions of energy quantities

No. of Superstructure Modes	**1**	**2**	**3**	**4**	**5**
Recoverable energy (kN-m)	10.412	9.994	10.035	10.068	10.065
Input energy (kN-m)	87.660	88.449	88.634	88.724	88.724

4.2 Effect of Geometry

To study the effect of geometry of VFPI, the variation of peak base shear and energy transmitted with $1/d$, also called the frequency variation factor (FVF), is plotted (fig. 4). Very low value of FVF makes the behaviour of the isolator similar to that of a FPS and for very high value the isolator behaves like a PF system. The spectra are plotted for 0.5*El Centro, 1.0*El Centro and 2.0*El Centro ground motion and coefficient of friction of 0.02. From the plots of response spectra it is found that there is a substantial reduction in the base shear and the energy transmitted with increase in FVF for high intensity excitation. This means the effectiveness of VFPI increases with higher variation of isolator frequency. However, with a very high value of FVF, VFPI may lose its restoring capability, and behave like PF system. It is therefore essential to choose the geometry such that there is sufficient reduction in the response without losing the restoring effect of the isolator.

4.3 Effect of Coefficient of Friction

The peak responses and energy quantities for different coefficients of friction of VFPI and FPS are tabulated in Table 4 and Table 5. These results are presented only for medium and high intensities of El Centro 1940 (NS) ground motion. The peak response and energy quantities have been normalised with respect to the corresponding peak quantities of the structure isolated using a FPS isolator. The results show that when compared with FPS, VFPI is quite effective in reducing the peak acceleration when the coefficient of friction is low. There is an increase in peak sliding displacements and residual displacements for increase in coefficients of friction. Although the sliding and residual displacement are higher for VFPI, they are within manageable limits. A substantial reduction in the base shear is also found. The use of VFPI does not show considerable reduction in input energy (except for low coefficient of friction and high intensity excitation) but shows a substantial reduction in the recoverable energy. This means that the FPS acts as an energy dissipator rather than an isolator. For lesser coefficients of friction, the FPS dissipates lesser energy thereby transmitting more energy into the structure. In case of VFPI, the performance depends on effective isolation at all levels and lesser energy dissipation does not affect its effectiveness.

TABLE 4 Effect of coefficient of friction on peak response of VFPI normalised to peak response values of FPS for El Centro ground motions (T_i=2s, T_b=2s)

Coefficient of friction	0.02		0.05		0.10	
Earthquake Intensity Factor	1	2	1	2	1	2
Absolute accl. Of top	0.7428	0.5000	0.9900	0.7391	0.9837	0.9523
Isolator displacement	1.5339	1.1167	1.0580	1.7446	1.4713	1.2012
Residual displacement	5.0878	1.9114	1.1364	1.3888	0.8147	1.2522
Base shear	0.4338	0.1856	0.7709	0.4935	0.9624	0.7171

TABLE 5 Effect of coefficient of friction on peak energy of VFPI normalised to peak energy values of FPS for El Centro ground motions (T_i=2s, T_b=2s)

Coefficient of friction	0.02		0.05		0.10	
Earthquake Intensity Factor	1	2	1	2	1	2
Recoverable energy	0.3534	0.0722	0.7865	0.3641	1.0110	0.7400
Absolute input energy	0.8204	0.4960	0.9487	0.8371	1.0056	0.9303

5. CONCLUSION

The effectiveness of a newly proposed sliding isolator namely, Variable Frequency Pendulum Isolator (VFPI) for vibration control of MDOF shear structures has been investigated. The following main conclusions can be drawn.

1. VFPI is very effective in reducing the response of a MDOF structure for different earthquake intensities..
2. The geometrical properties of the VFPI can be determined to suit the requirements of a given problem.
3. Only first few modes of the superstructure need to be considered in the analysis.
4. VFPI behaves like an effective isolator for any input excitation, whereas FPS behaviour is primarily as an energy dissipator.

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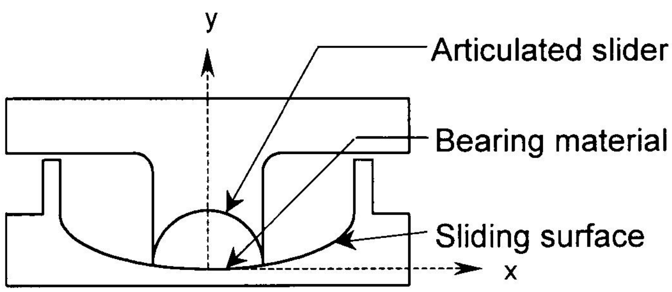

Fig. 1 Friction Pendulum System

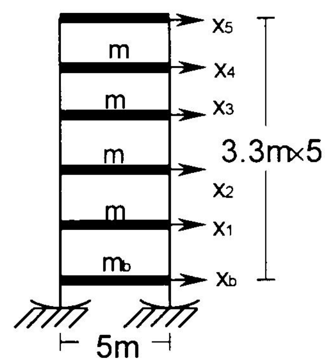

Fig. 2 Shear building on VFPI

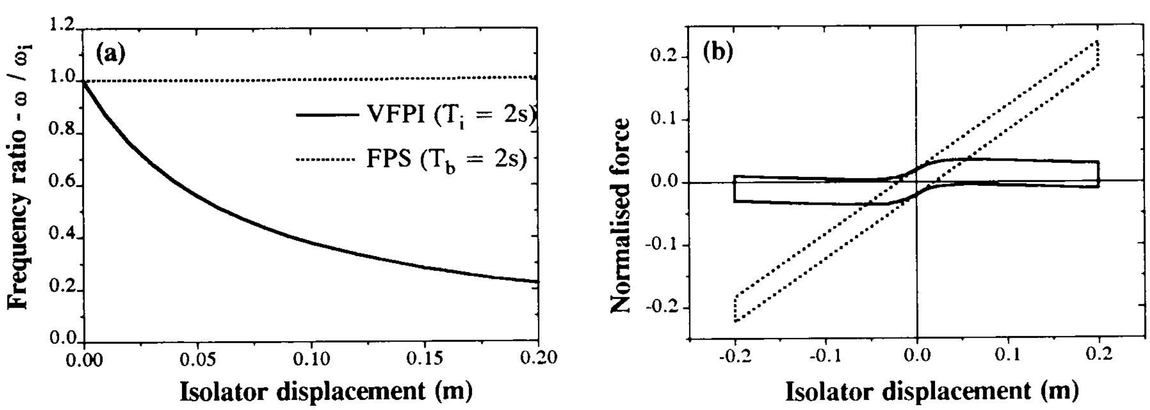

Fig. 3 Properties of VFPI and FPS (a) Frequency variation (b) Hysteresis loops.

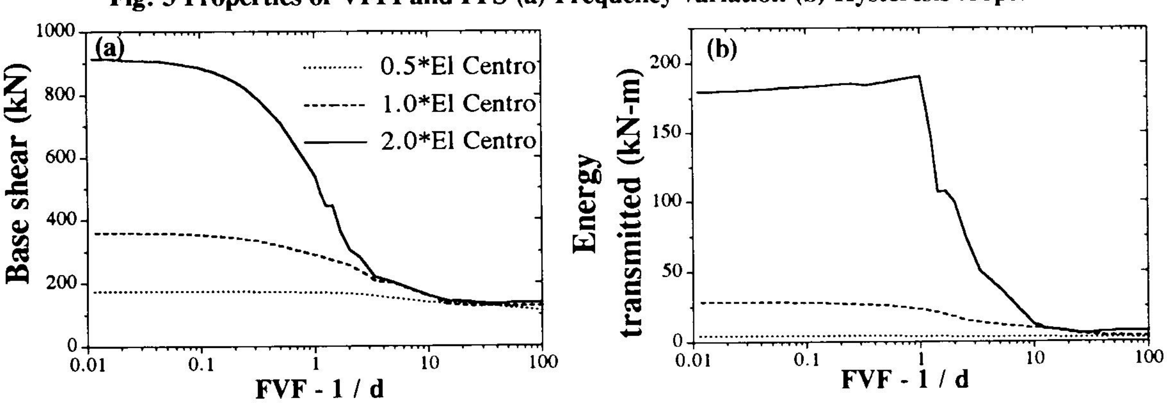

Fig. 4 Effect of geometry on example system subjected to scaled intensities of El Centro 1940 (NS) ground motion (a) Base shear (b) Energy transmitted to superstructure (T_i = 2s, μ = 0.02, ξ = 5%).